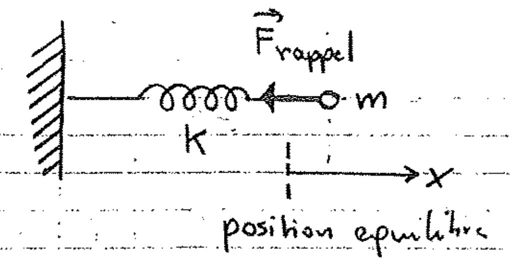
Mouvement périodique d'une masse = oscillateur



$\sum \vec{F} = m\vec{a}$

$\underbrace{\vec{P} + \vec{R}}_{\vec{0}} + \vec{F}_{rappel} = m\vec{a}$ sans frottement

$\longrightarrow$ Pb à 1 dimension

$m\vec{a} = \vec{F}_{rappel} \longrightarrow m \dfrac{d^2x(t)}{dt^2}\vec{e_x} = -kx\,\vec{e_x}$

$\dfrac{d^2x(t)}{dt} + \dfrac{k}{m}x(t) = 0 \qquad \dfrac{d^2}{dt^2}x(t) + \omega_0^2 x(t) = 0 \qquad \boxed{\omega_0^2 = \dfrac{k}{m}}$

A.N. $m = 0{,}1\,kg \quad K = 10\,kg.s^{-2} \quad \omega_0 = \sqrt{\dfrac{10}{0{,}1}} = 10\ rad.s^{-1}$

éq. caractéristique $e^{rt} \rightarrow r^2 + \omega_0^2 = 0 \qquad r = \pm i\omega_0$

$x(t) = A e^{i\omega_0 t} + B e^{-i\omega_0 t}$ combinaison linéaire

conditions initiales. $x(t=0) = x_0 \qquad \dfrac{dx}{dt}(t=0) = 0$

$x(t=0) = A + B = x_0$

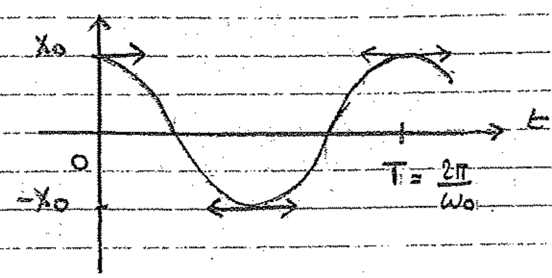
$\dfrac{dx}{dt}(t) = i\omega_0\left(Ae^{i\omega_0 t} - Be^{-i\omega_0 t}\right) \qquad \dfrac{dx}{dt}(t=0) = i\omega_0(A-B) = 0 \longrightarrow A = B = \dfrac{x_0}{2}$

solution $x(t) = \dfrac{x_0}{2}\left(e^{i\omega_0 t} - e^{-i\omega_0 t}\right) = \boxed{x_0 \cos\omega_0 t}$

on peut montrer qu'on obtient le même résultat avec $x_1(t)$, $x_2(t)$ et $x_3(t)$)

$x_1(t) \longrightarrow A_1 = x_0, \; B_1 = 0 \qquad x_2(t) \rightarrow A_2 = x_0, \; \varphi_2 = 0$

$x_3(t) \longrightarrow A_3 = x_0 \quad \varphi_2 = \dfrac{\pi}{2}$



$\boxed{T = \dfrac{2\pi}{\omega_0}}$